Solve System of Initial Value ODEs Using Explicit Euler Method

Objectives

- Solve a System of First order ODE using Explicit Euler Method
- A second or higher order ODE can be converted to a system of first order ODEs and can be solved using this procedure.
- The methods for solving a single first-order ODE can be used to solve systems of coupled first order ODEs.

System of ODEs – Explicit Euler Method

- Consider a system of general nonlinear first order ODEs of the form
- $y1' = f1(t, y1, y2)$, $y1(t_0) = y1_0$ .....(1)
- $y2' = f2(t, y1, y2)$, $y2(t_0) = y2_0$ .....(2)
- $y1_{n+1} = y1_n + \Delta t * f1_n$ .....(3)
- $y2_{n+1} = y2_n + \Delta t * f2_n$ .....(4)

System of ODEs – Explicit Euler Method

- Example ODE Problem
- $\frac{dy1}{dx}$ = - 0.5 * y1
- $\frac{dy2}{dx}$ = 4 - 0.3 * y2 – 0.1 * y1
- From x = 0 to 2 with a step size of 0.5;
- The initial condition at *x* = 0 is *y1* = 4, y2 = 6.

System of ODEs – Explicit Euler Method

- $y1_{n+1} = y1_n + \Delta t * f1_n$ (Note Independent variable can be x or t)
- $f1_n = -0.5 * y1_n$
- Let n = 0;
- $y1_1 = y1_0 + \Delta t * f1_0$
- $f10 = -0.5 * y1_0 = -0.5 * 4 = -2$
- $y1_1 = y1_0 + \Delta t * f10 = 4 + 0.5 * (-2) = 3$

System of ODEs – Explicit Euler Method

- $y2_{n+1} = y2_n + \Delta t * f2_n$ (Note Independent variable can be x or t)
- $f2_n = 4 - 0.3 * y2_n - 0.1 * y1_n$
- Let n = 0;
- $y2_1 = y2_0 + \Delta t * f2_0$
- $f2_0 = 4 - 0.3 * y2_0 - 0.1 * y1_0 = 4 - 0.3 * 6 - 0.1 * 4 = 1.8$
- $y2_1 = y2_0 + \Delta t * f2_0 = 6 + 0.5 * (1.8) = 6.9$
- Likewise, $y1_2$, $y2_2$ $y1_3$, $y2_3$ etc can be evaluated

Summary

In this video,

- We presented Explicit Euler Method to solve a system of Initial Value ODEs
- The FDE is explicit, since f_n does not depend on y_{n+1}
- The explicit Euler method is conditionally stable.
- The global error is 0(Δt).
- The error can be minimized by using smaller steps.
- In the next video we can look at Fourth Order Runge−Kutta Method to solve a system of Initial Value ODEs